Optimization of thermoacoustic engine driven thermoacoustic refrigerator using response surface methodology

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Optimization of thermoacoustic engine driven thermoacoustic refrigerator using response surface methodology

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Abstract. Thermoacoustic engines (TAEs) are devices which convert heat energy into useful acoustic work whereas thermoacoustic refrigerators (TARs) convert acoustic work into temperature gradient. These devices work without any moving component. Study presented here comprises of a combination system i.e. thermoacoustic engine driven thermoacoustic refrigerator (TADTAR). This system has no moving component and hence it is easy to fabricate but at the same time it is very challenging to design and construct optimized system with comparable performance. The work presented here aims to apply optimization technique to TADTAR in the form of response surface methodology (RSM). Significance of stack position and stack length for engine stack, stack position and stack length for refrigerator stack are investigated in current work. Results from RSM are compared with results from simulations using Design Environment for Low- amplitude Thermoacoustic Energy conversion (DeltaEC) for compliance.

1. Introduction

As the name thermoacoustics suggests, it involves conversion of heat energy and acoustic energy in one another. Thermoacoustics has become area of interest for many researchers due to the advantages of thermoacoustic technology like absence of moving components making devices more reliable and less maintenance prone, constructional simplicity, usability of noble gases and low grade energy sources, structural stability. Though the theory of thermoacoustics is well established there is no simple approach available for design of thermoacoustic devices. A quantitative engineering approach to design of thermoacoustic refrigerator (TAR) is given by Tijani et al. [1]. More detailed explanation on working of thermoacoustics is given by Swift [2].

Thermoacoustic devices mainly consists four components. A hot heat exchanger, a stack – often called the heart of thermoacoustic device, a cold heat exchanger and a resonator tube. Thermoacoustic engines develop acoustic power by using heat energy. Acoustic oscillations are generated due to the thermal interaction between the oscillating gas and the surface of the stack. The heat exchangers exchange heat with surroundings and maintain much required temperature gradient along the length of the stack for generation of acoustic work whereas thermoacoustic refrigerator uses acoustic energy to produce cooling effect. In figure 1 a simple illustration of thermoacoustic engine and in figure 2, a simple illustration of thermoacoustic refrigerator is shown.

It is peculiar for thermoacoustic systems that performance of these systems is very sensitive to physical dimensions and operating conditions. Minor change in combination of geometric parameter and operating parameter affects the performance steeply. Due to this it is very much required to trace the effect of different parameter for required output and to categorize significant parameter for particular performance output.
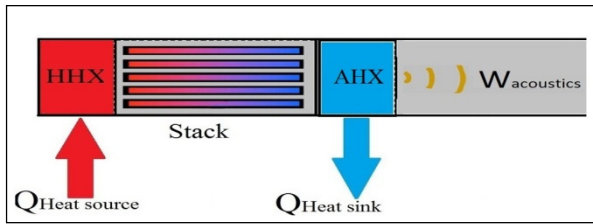
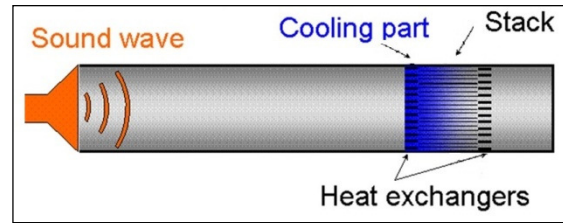**Figure 1.** A Simple illustration of thermoacoustic engine.

Figure 2. A Simple illustration of thermoacoustic refrigerator

In current work, Response surface methodology (RSM) is used to trace significant parameters affecting a performance output of thermoacoustic engine driven thermoacoustic refrigerator system. Stack length (L) and stack position (X) of engine and refrigerator are investigated as variables affecting performance of the system. Then using same variables, simulations are performed in DeltaEC to match RSM model for compliance.

2. Thermoacoustically driven thermoacoustic refrigerator (TADTAR)

Thermoacoustic engine develops acoustic oscillations while thermoacoustic refrigerator require acoustic oscillations for working. In TADTAR system, TAE and TAR are combined as a single system and acoustic energy produced by TAE is used by TAR to produce cooling effect. By using this system, cold can be produced by using heat. It is also advantageous with the fact that waste heat can be utilized as a source of energy to produce cooling. TADTAR system mainly consists of a hot heat exchanger to supply heat to the system, a TAE stack which will produce acoustic power, an ambient heat exchanger on TAE side to exchange heat with surrounding maintaining that end of stack to room temperature, a resonator pipe to sustain acoustic oscillations, a cold heat exchanger to accumulate cooling effect, a stack for TAR part and finally an ambient heat exchanger on TAR side to maintain that end of TAR stack at room temperature. Figure 3 shows a schematic of TADTAR system. It also shows how length and position of stack are defined.
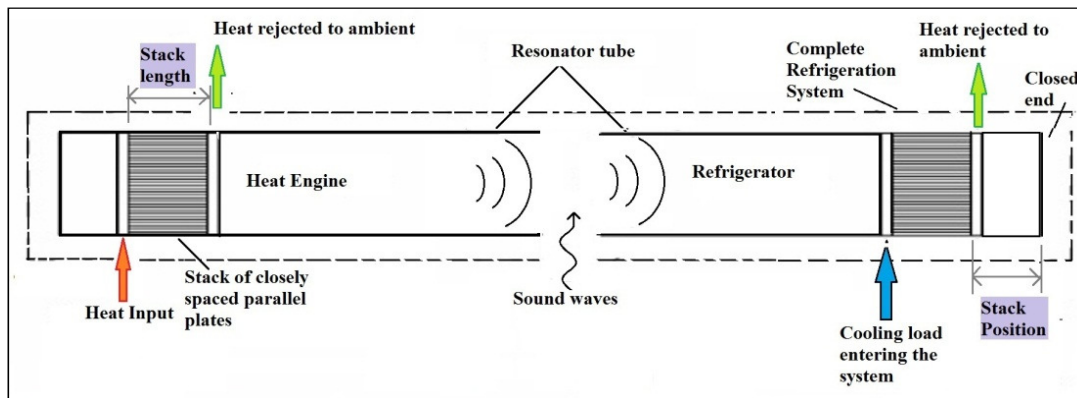

Figure 3. A Simple illustration of thermoacoustic engine driven thermoacoustic refrigerator

3. Response surface methodology (RSM)

Design methodology for thermoacoustic engine highlighted by the authors [3] involves many independent and dependent parameters affecting performance of the device. Higher numbers of parameters increase number of simulative experiments as well as actual experiments in great way.

To reduce the number of experiments, optimization using Response surface methodology (RSM) has been incorporated in many areas e.g. electrochemical treatment processes [4], sheet metal forming process [5], electro discharge machining process [6, 7] etc. Optimization of TAR [8] and TAE [9] using RSM is also cited but there is no literature showing RSM optimization approach being used for a combined thermoacoustic system viz. TADTAR. Therefore main objective of this work is to understand and apply RSM optimization to TADTAR system and list out significant variable among the list for a particular response.

A combine system includes both, a thermoacoustic engine part and a thermoacoustic refrigerator part, and hence involves more complexities as far as parametric optimization is concerned. So to start

with, Length and position of engine and refrigerator stack are considered as variable for RSM. Cooling power produced by TAR stack and acoustic power produced by TAE stack are considered as responses.

RSM is a platform which uses mathematical rules and statistical techniques to identify the effect of independent variables alone or their effect in combination. This behavior of variables is checked on the output, also called response, by generating a mathematical model. At first RSM identifies the independent parameters and their respective levels then it verifies the generated model with actual or simulative experiments and finally it plots graphs. RSM distributes experimental points and randomizes experimental errors of each run and hence its accuracy is increased. Over and above this with RSM it becomes possible to check performance at intermediate levels with the help of mathematical model which ultimately leads to parametric investigation path.

The relation between these variables and respective response is given by following equation.

$$y = f(x_1, x_2, x_3 \dots x_k) + \varepsilon \tag{1}$$

Where, y is response, f is the unknown function of response, $x_1, x_2, x_3 \dots x_k$ are independent variable, k is number of independent variable and ε is statistical error. In current work RSM processing is carried out in Minitab 17, a Minitab, Inc product, using second order central composite (CCD) design. This second order model provides a correlation between variables and response which can be given as follows.

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i=1}^{k} \sum_{j=i+1}^{k} \beta_{ij} x_i x_j + \varepsilon \tag{2}$$

Where β_0, β_i, β_{ii}, and β_{ij} are regression coefficients for the intercept, linear, quadratic and interaction coefficients, respectively, and x_i and x_j are the coded independent variables [9]. A four-factor five level CCD is considered for current work of optimization of TADTAR and 31 runs were performed in random order. Using this factors and levels, information is generated to fit a second order polynomial. Considering the effects of these variables, individually and in combination a regression equation as shown by equation 3 is formed.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{33} x_3^2 + \beta_{44} x_4^2 + \beta_{55} x_5^2 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{14} x_1 x_4 + \beta_{15} x_1 x_5 + \beta_{23} x_2 x_3 + \beta_{24} x_2 x_4 + \beta_{25} x_2 x_5 + \beta_{34} x_3 x_4 + \beta_{35} x_3 x_5 + \beta_{45} x_4 x_5 \tag{3}$$

Table 1. Range and level of independent variable in CCD

Variable (cm)	Range and levels				
	-2	-1	0	1	2
Xe, stack position of TAE	10.0	12.5	15.0	17.5	20.0
Le, stack length of TAE	3.25	4.50	5.75	7.00	8.25
Xr, stack position of TAR	3.500	5.875	8.250	10.625	13.000
Lr, stack length of TAR	2.5000	3.9375	5.3750	6.8125	8.2500

In experimental design model, stack position of TAE (10 cm-20 cm), stack length of TAE (3.25 cm-8.25 cm), stack position of TAR (3.5 cm-13 cm) and stack length of TAR (2.5 cm-8.25cm) were considered as independent variables whereas cooling power produced by TAR stack, Q_{cr} and acoustic power produced by TAE stack, W_{pro} were considered as responses. The range and levels of independent variables investigated in current work are as per table 1.

The experimental design matrix resulting from the CCD model is shown in table 2. First column of table 2 shows run number, the next column shows Variable (X_e), third column shows variable (L_e), the next column shows variable (X_r), fifth column shows variable (L_r). Sixth and seventh column shows DeltaEC and RSM values of Q_{cr} respectively. Last two columns show DeltaEC and RSM values of W_{pro} respectively.

4. DeltaEC modeling

DeltaEC (Design environment for low amplitude thermoacoustic energy conversion) is great tool to check and predict performance of thermoacoustic devices. It is freely available thanks to G.W swift

and Los Alamos National Laboratory. DeltaEC numerically integrate one dimensional wave equation to user defined geometry. It uses segment such as DUCT, STKSLAB, and HX etc. to create a model.

Table 2. Experimental design matrix and RSM results for responses

Run	Factors (cm)				Response			
	Xe	Le	Xr	Lr	Cooling power produced by TAR stack (W)		Acoustic power produced by TAE stack (W)	
					DeltaEC	RSM	DeltaEC	RSM
1	17.5	4.50	10.625	3.9375	58.07	57.45	180.27	180.21
2	12.5	4.50	10.625	3.9375	58.07	57.48	129.66	129.71
3	15.0	5.75	8.250	5.3750	77.22	77.10	152.65	152.64
4	15.0	5.75	8.250	5.3750	77.22	77.10	152.65	152.64
5	10.0	5.75	8.250	5.3750	77.22	77.57	101.43	101.35
6	12.5	7.00	10.625	3.9375	58.07	57.47	126.38	126.41
7	17.5	7.00	10.625	3.9375	58.07	57.44	175.59	175.52
8	12.5	7.00	10.625	6.8125	90.67	91.76	125.12	125.06
9	17.5	4.50	5.875	3.9375	59.26	57.50	180.22	180.23
10	15.0	8.25	8.250	5.3750	77.22	77.53	148.07	148.07
11	12.5	7.00	5.875	6.8125	71.07	71.02	125.88	125.89
12	15.0	5.75	13.000	5.3750	74.12	72.12	152.78	152.90
13	17.5	4.50	10.625	6.8125	90.67	91.75	178.83	178.73
14	20.0	5.75	8.250	5.3750	77.22	77.52	200.69	200.83
15	15.0	5.75	3.500	5.3750	48.78	51.42	153.82	153.76
16	17.5	7.00	10.625	6.8125	90.67	91.74	174.00	173.94
17	15.0	5.75	8.250	5.3750	77.22	77.10	152.65	152.64
18	12.5	4.50	5.875	3.9375	59.26	57.52	129.62	129.64
19	12.5	7.00	5.875	3.9375	59.26	57.51	126.33	126.38
20	15.0	5.75	8.250	5.3750	77.22	77.10	152.65	152.64
21	17.5	7.00	5.875	6.8125	71.07	70.99	174.96	174.86
22	15.0	5.75	8.250	2.5000	33.24	36.97	154.45	154.34
23	17.5	4.50	5.875	6.8125	71.07	71.01	179.70	179.62
24	17.5	7.00	5.875	3.9375	59.26	57.48	175.53	175.58
25	15.0	5.75	8.250	5.3750	77.22	77.10	152.65	152.64
26	15.0	5.75	8.250	5.3750	77.22	77.10	152.65	152.64
27	15.0	5.75	8.250	5.3750	77.22	77.10	152.65	152.64
28	12.5	4.50	10.625	6.8125	90.67	91.77	128.56	128.46
29	12.5	4.50	5.875	6.8125	71.07	71.03	129.23	129.25
30	15.0	5.75	8.250	8.2500	87.86	84.77	152.21	152.38
31	15.0	3.25	8.250	5.3750	77.22	77.55	156.06	156.12

To generate a TADTAR model in DeltaEC, it requires segments such as BEGIN, SURFACE, DUCT, HX, STKSLAB and HARDEND. Helium gas was selected as a working fluid whereas stainless STKSLAB was used as TAE stack and mylar STKSLAB as TAR stack material. Stainless DUCT was used as resonator. Copper was used as HX material for all four heat exchanger.

Figure 4 shows DeltaEC generated model of TADTAR. In this figure, component number 3, 5, 7 and 9 are heat exchangers. Component number 4 and 8 are TAE stack and TAR stack respectively. In this TADTAR configuration it is assumed that a TAE and a TAR of quarter wave type are combined together to form a half wave type TADTAR system and to simulate the same in model a HARDEND is shown at the end. After generating DeltaEC model, the experimental conditions of design matrix

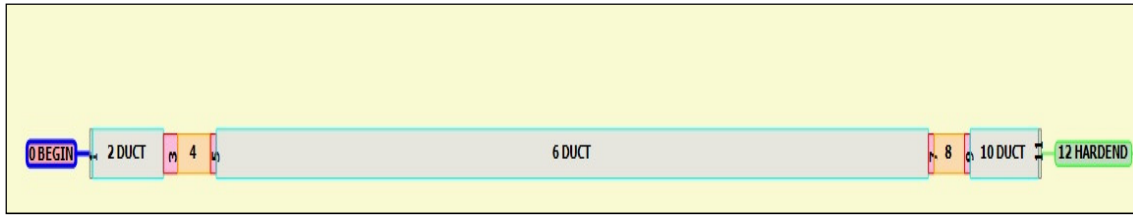are taken as input to the DeltaEC model and output is extracted as shown in column number 6 and 8 of table 2.

Figure 4. DeltaEC model of TADTAR

5. Results and discussion

Using output values of responses as per table 2, RSM generates a mathematical model in terms of input variables. The values of Q_{cr} are calculated from following RSM generated model.

$$Q_{cr} = -14.7 - 0.53X_e - 0.81L_e + 5.21X_r + 16.87L_r + 0.0175X_eX_e + 0.07L_eL_e - 0.6795X_rX_r - 1.964L_rL_r + 0X_eL_e - 0X_eX_r - 0X_eL_r + 0L_eX_r - 0L_eL_r + 1.522X_rL_r \qquad (4)$$

Upon generating this model it is required to be test for variable significance and adequacy of the model and for this ANOVA analysis is used. In ANOVA the sum of squares is used to approximate the mean square of deviation from grand mean. The mean squares are obtained by dividing sum of squares by degrees of freedom. The fisher variation ratio is estimated by dividing means square by error mean square. Larger the fisher ratio better the model can be explained by derived regression equation. Similarly if p-values is lesser than 0.05, generated model is statistically significant. It explains that variable selected for tracing response have significant effect on response. Likewise if a p-value for a variable or a combination of variables is less than 0.05 then it indicates that the variable is significant for that particular response. P-values less than 0.05 indicate that the model is statistically significant at 95% of probability level. P-values greater than 0.05 indicated that the variables are not significant and their variation can be ignored for a response. R-squared values being displayed at the bottom of ANOVA table are defined as ratio of the explained variation to the total variation. If these values approach unity it indicates good relationship between actual values and values predicted by RSM. In our case actual values are the values given by DeltaEC model.

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	14	5351.31	382.24	115.26	0.000
Linear	4	4073.15	1018.29	307.05	0.000
xe	1	0.00	0.00	0.00	1.000
le	1	0.00	0.00	0.00	1.000
xr	1	643.98	643.98	194.18	0.000
lr	1	3429.17	3429.17	1034.01	0.000
Square	4	845.94	211.49	63.77	0.000
xe*xe	1	0.34	0.34	0.10	0.752
le*le	1	0.34	0.34	0.10	0.752
xr*xr	1	420.11	420.11	126.68	0.000
lr*lr	1	470.88	470.88	141.99	0.000
2-Way Interaction	6	432.22	72.04	21.72	0.000
xe*le	1	0.00	0.00	0.00	1.000
xe*xr	1	0.00	0.00	0.00	1.000
xe*lr	1	0.00	0.00	0.00	1.000
le*xr	1	0.00	0.00	0.00	1.000
le*lr	1	0.00	0.00	0.00	1.000
xr*lr	1	432.22	432.22	130.33	0.000
Error	16	53.06	3.32		
Lack-of-Fit	10	53.06	5.31	*	*
Pure Error	6	0.00	0.00		
Total	30	5404.38			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
1.82109	99.02%	98.16%	94.34%

Figure 5. ANOVA table for cooling power produced by TAR

Analysis of Variance **for Acoustic power produced**

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	14	14956.5	1068.3	116484.83	0.000
Linear	4	14946.4	3736.6	407421.89	0.000
xne	1	14842.4	14842.4	1618345.10	0.000
lsne	1	97.1	97.1	10589.85	0.000
xnr	1	1.1	1.1	120.03	0.000
lsnr	1	5.8	5.8	632.59	0.000
Square	4	7.3	1.8	199.54	0.000
xne*xne	1	4.3	4.3	470.70	0.000
lsne*lsne	1	0.5	0.5	58.77	0.000
xnr*xnr	1	0.8	0.8	91.66	0.000
lsnr*lsnr	1	0.9	0.9	99.86	0.000
2-Way Interaction	6	2.8	0.5	50.32	0.000
xne*lsne	1	1.9	1.9	212.19	0.000
xne*xnr	1	0.0	0.0	0.98	0.336
xne*lsnr	1	0.1	0.1	5.77	0.029
lsne*xnr	1	0.0	0.0	0.17	0.682
lsne*lsnr	1	0.0	0.0	1.20	0.289
xnr*lsnr	1	0.7	0.7	81.58	0.000
Error	16	0.1	0.0		
Lack-of-Fit	10	0.1	0.0	*	*
Pure Error	6	0.0	0.0		
Total	30	14956.7			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0957672	100.00%	100.00%	99.99%

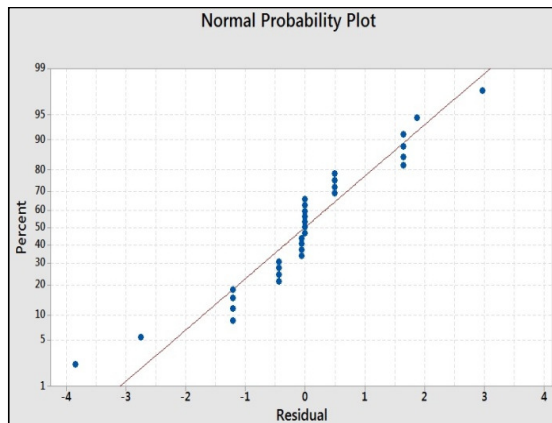**Figure 6.** ANOVA table for acoustic power produced by TAE

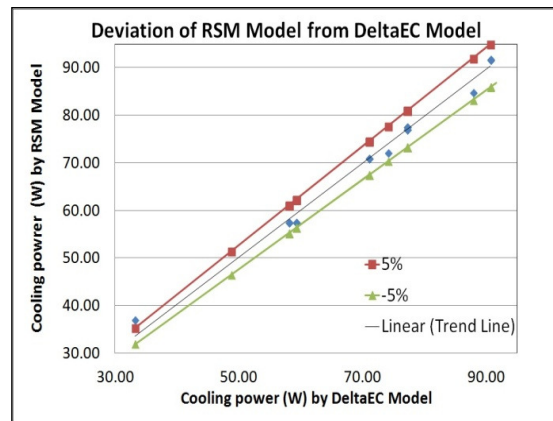**Figure 7.** Normal probability chart for cooling power

Figure 8. Deviation chart for cooling power

Figure 5 shows ANOVA table for cooling power. As it can be seen that for model of cooling power, fisher ratio is higher which indicates that our model is significant. Also corresponding p-value is also less than 0.05 which indicates variables under investigations are significant for cooling power. More over p-values of TAR stack position and TAR stack length as well as their interaction are less than 0.05 which point out that these two variables are most significant for cooling power. On the other hand TAE stack position and TAE stack length are having higher p-values so as their interaction. It means that these two variables are non-significant variable for cooling power as response.

On the similar note, RSM generated model for W_{pro} as shown in equation 5 and ANOVA table for acoustic power produced by TAE stack as shown in figure 6. It can also be analyzed like cooling power table. Seeing the table it can be observed that model is significant. Moreover all four variables under investigation are also significant for acoustic power. Also R-squared values for both responses approaching unity which means actual and predicted values are comparable to each other.

$$W_{pro} = -11.88 + 12.573X_e + 1.181L_e - 0.172X_r - 0.426L_r - 0.06217X_eX_e - 0.0879L_eL_e + 0.0304X_rX_r + 0.0866L_rL_r - 0.1116X_eL_e - 0.004X_eX_r - 0.016X_eL_r - 0.00337L_eX_r - 0.0146L_eL_r - 0.06334X_rL_r \tag{5}$$

Figure 7 shows normal probability chart for cooling power. It indicates that errors are normally distributed for response as all the data points lay near diagonal line. Figure 8 shows deviation chart for cooling power. It displays relationship between actual values, values given by DeltaEC in our case and RSM predicted values. It shows that predicted values are within 5% range of actual values and shows a good association between actual model and RSM generated model. When cooling power model is optimized to give maximum cooling power, model gave TAE stack position X_e as 14.9 cm, TAE stack length L_e as 3.2 cm, TAR stack position X_r as 11.6 cm, TAR stack length L_r as 7.6 cm and cooling power as 97.37 W. Upon checking these variable values in DeltaEC it gave cooling power as 95.52 W. Variation in the values is only 1.9% which again shows model's significance. This same exercise was carried out for acoustic power as well and it also showed good model significance.

6. Conclusions
To investigate the performance of a thermoacoustic engine driven thermoacoustic refrigerator, where many parameters are complicatedly involved, response surface methodology turn out to be a great tool of optimization. Using the RSM, an effective mathematical model has been developed for optimization of TADTAR system. The effect of variables like stack position of TAE, stack length of TAE, stack position of TAR and stack length of TAR on the performance of system i.e. cooling power produced by TAR stack and acoustic power produced by TAE stack has been investigated and variables are optimized. The values of F- test, p-test, and Adeq precision from ANOVA table showed that the quadratic model generated by RSM for the responses are significant. ANOVA analysis of model was carried out wherein variables and their interactions were studied to identify significant variables. The result obtained from RSM disclosed that position and length of TAR stack are only significant variable for cooling power produced by TAR stack whereas all four variables under

investigation are having significant effect on acoustic power produced by TAE stack. Simulations were carried out for RSM produced values of all four variables under investigation and results showed good association with RSM values. The present work can be extended by assuming other variables as one of the factor and experimental verification can also be carried out..

8. References
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